Running on rough terrains: Energetics and stability

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1 Introduction

How do animals run stably on uneven terrain? Multiple strategies to improve stability, such as increasing rate of swing leg retraction and increasing step width have been proposed as well as shown to correlate with experimental data [1–3]. However, these incur a substantial additional energetic cost [4–6]. Recent experiments [7, 8] have measured energy consumption for uneven terrain walking and running and found that the increase in energy expenditure (compared to flat ground) for walking is 28% (0.73 W/kg), while for running it is 5% (0.68 W/kg).

We ask why the energy consumption to run on uneven terrain is comparable to, and not much larger than, level ground. Using results from our bouncing mass model [9] we show that the typical energy dissipation for humans running on flat terrain is sufficient to remain stable for over 200 steps using a nearly open-loop strategy. Therefore, running on rough terrain, without falling, is nearly as easy as running on flat terrain. These are also consistent with experimental data we collected from speed-controlled overground running experiments on uneven terrains.

2 Model and experiments

2.1 Bouncing mass model for running

We use the bouncing mass model for running, presented earlier at Dynamic Walking [9], but with one important modification. We define a grid along the x-axis with spacing \( \lambda \) (here we use a grid spacing of 0.5\( r_{ball} \)). We then choose a random number for the terrain height from a specified distribution, at each of the grid points. The terrain is then represented as a piecewise linear and continuous curve. The probability distribution used in our simulations is the uniform distribution over the interval \( \pm 0.05 r_{ball} \).

Drag-free projectile equations describe the ball’s flight phase. We find the collision of the ball with the terrain using custom code written in C. Figure 1 shows a schematic of the model and possible failure modes. The collision with the ground and active push-off are both identical to our previous work [9]. The collision is modeled using coefficients of restitution in 2D (normal and tangential to the terrain), and the active-push-off is blind to the state or the terrain with constant impulse magnitude and direction in the lab frame. In that sense, this model uses an open-loop strategy. Sensors detect contact, and determine when to apply an impulse against the ground, the active push-off. The constant impulse is chosen such that the bouncing mass will execute a perfectly periodic trajectory on perfectly flat ground. In cases where the tangential collision is also inelastic, we include a torque-impulse term that would maintain a constant orientation on flat ground.

2.2 Uneven terrain experiments

We conducted overground running experiments on custom-made uneven terrains and on flat ground while simultaneously recording breathing gas data (Oxycon mobile, Carefusion Inc.), kinematics and kinetics from 10 subjects (8 men, 2 women). Experiments were conducted at the National Cen-
tre for Biological Sciences (NCBS) with informed consent from the volunteers, and approval from the NCBS’ IRB. All subjects were regular half-marathon or marathon runners, and ran an average of at least 30 kilometers/week. Subjects were asked to run at 2.91m/s on all terrains (figure 1). Each run lasted for at least 10min where subjects ran back and forth on the 25m track, staying within the light band, which controlled the speed and allowable fluctuation in speed.

Figure 2. a. Average and standard deviations of the oxygen consumption for all runners for each terrain type. Energy consumption on the ‘Roughigh’ terrain is significantly different from ‘Flat’ (Wilcoxon Rank test) b. Median number of steps before failure is plotted as a function of the square of the normal coefficient of restitution \( n^2 \) for 3 running speeds (tangential coefficient of restitution \( \varepsilon_r = 1 \)). \( 1 - \varepsilon_r^2 \) is the energy dissipated by the passive collision. Simulations were run for a maximum of 1000 steps with an ensemble size of \( 10^6 \). c. Contour plot of median number of steps steps traveled as a function of speed and \( \varepsilon_n^2 \). Humans typically elastically store around 40% of the energy per step during running, which corresponds to \( \varepsilon_n^2 \sim 0.4 \). At these dissipation levels and speeds probed in our experiments or by [8], the open-loop bouncer travels around 200 steps before failure. This is far greater than typical neural feedback time scales (\( \sim 1000 \times \)).

Data from the first 3 minutes of the run were not included in the analysis to allow for transients to stabilize. We report the time-averaged \( O_2 \) consumption (metabolic power) normalized by mass and target running speed (speed of lights), i.e. a mass and distance normalized metabolic energy estimate (the cost of transport).

### 3 Results

Figure 2 shows that, like reported in [8], increase in oxygen consumption for uneven terrain running is low, only about 5.8% higher than for the flat terrain. Note that [8] report power consumption, while we show energy consumption per unit distance, the cost-of-transport. The difference in energy consumption between the low roughness and flat terrain was not statistically different. Simulations of our model show that the open-loop bouncing mass can traverse as many as 200 steps before failure, at speeds typical of endurance runners. In order to determine why this strategy does so well, we performed a linear stability analysis of the ball on flat terrain by calculating the return map from one touchdown to the next in a frame translating forward along with the ball. The map was linearized about an arbitrary forward running speed. The eigenvalues of the system are 1, 1, 1, \( \varepsilon_r, \varepsilon_n \). Therefore, to a linear approximation, instabilities that exist do not grow exponentially, rather they grow polynomially with the number of steps.

### 4 Conclusions

Analyses of running, as a bouncing gait with intermittent and impulsive ground contact, suggest that neglecting the unevenness of the terrain does not lead to drastically unstable gaits, i.e. instabilities do not grow exponentially. Importantly, we do not incorporate any feedback to correct for perturbations from the uneven terrain. The only use of sensory feedback is to apply ground impulses when in contact with the terrain. We therefore propose that occasional feedback corrections, with delays of several steps even, may suffice to remain stable on uneven terrains. This may underlie the surprisingly small increase in energy consumption for running on rough terrains. However, why the marginal cost of walking on rough terrain is almost as expensive as running remains unclear and the topic of ongoing work.

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### References


