Reduced-Order Dynamic Model in 3-D Cartesian Space for the Construction of Bipeds’ Balanced Domain

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1 Introduction

Bipeds maintain balance by regulating angular momentum \((H_{COM})\), redirecting ground reaction forces \((GRF)\), and changing center of pressure \((COP)\) location. If a biped’s trajectory passing through a certain state terminates in a fallen configuration, no matter what control law is used, then that state is called falling; vice versa, the state is called balanced (Fig. 1, [1]). Initial conditions, unilateral constraints, and other system’s limitations (e.g., size of foot support region, FSR, and joint angle limits) impose a boundary to the domain of feasible and balanced state trajectories of a biped system. Existing approaches utilize a specific control law, i.e., \textit{a priori} choice of controller design and parameters, to identify such boundary.

![Figure 1: Balanced, falling, and fallen domain [1].](image1)

Here, the identification of a general balanced state domain, as the space representing the physically feasible and reachable superset of all possible controller-specific domains, is addressed. A general construction framework that provides 3-D balance criteria for general multisegmental robots is introduced, based on a reduced-order dynamic model. The proposed model includes a 3-D mechanism with minimal number of degrees of freedom \((DOFs)\) and is capable of representing the equivalent center-of-mass \((COM)\) dynamics of any biped under any task. A numerical optimization algorithm is established to solve for the minimum and maximum Cartesian \(COM\) velocity components of the 3-D mechanism, which provides the boundaries of the balanced state manifold as the criteria of balanced versus falling of the real legged robot.

2 Dynamic Model

The \(COM\) dynamics of a generic biped system (Fig. 2) is governed by the translational and rotational EOM written in the \(COM\) 3-D Cartesian space:

\[
\begin{align*}
GRF + mg &= m\dot{r}_{COM} \\
(r_{COP} - r_{COM}) \times GRF + M_{COP}^{GRF} &= \dot{H}_{COM}
\end{align*}
\]

where \(g=[0, -9.807, 0]^T\), \(m\) is total mass, \(r_{COM}\) and \(r_{COP}\) are \(COM\) and \(COP\) position vectors with respect to inertial frame \((O, X, Y, Z)\), and \(H_{COM}\) is the rate of change of the angular momentum about \(COM\). The left-hand side of Eq. (2) is rearranged using an equivalent force-couple system \(M_{O}^{GRF} = M_{COP}^{GRF} + r_{COP} \times GRF\) at contact point \(O\):

\[
-r_{COM} \times GRF + M_{O}^{GRF} = \dot{H}_{COM}
\]

where \(M_{COP}^{GRF}\), with zero tangential components, and \(M_{O}^{GRF}\) are the resultant ground reaction moment due to contact forces distribution about \(COP\) and \(O\), respectively.

![Figure 2: Dynamic model of generic biped system.](image2)

Based on Eqs. (1) and (3), a reduced-order dynamic model is developed (Fig. 3) that can describe the same \(COM\) dynamics of the real multibody system, with reduced complexity and without loss of generality. The structure of the reduced-order dynamic model is a 3-DOF prismatic mechanism (Fig. 3) with a single mass \(m\) coincident with the real biped system’s \(COM\). Variables \(q = [q_1, q_2, q_3]^T\) represent the three independent generalized coordinates describing the \(COM\) motion in 3-D Cartesian space.

Since the same external forces are applied to the real biped and to the reduced-order mechanism, the two systems are dynamically equivalent with respect to \(COM\) translation. On the other hand, the reduced-order model does
not have a rotational response about the COM. To compensate for the effects of $\mathbf{H}_{\text{COM}}$ existing in the real biped system, an additional external couple $\mathbf{C}_{\text{COM}} = -\mathbf{H}_{\text{COM}}$ is applied to the prismatic mechanism’s COM, yielding to:

$$-r_{\text{COM}} \times \mathbf{GRF} + \mathbf{M}_{\text{REF}} + \mathbf{C}_{\text{COM}} = 0$$

(4)

In general, $\mathbf{H}_{\text{COM}}$ is limited by its lower and upper bounds, $\mathbf{H}_{\text{COM}}^{\text{LB}}$ and $\mathbf{H}_{\text{COM}}^{\text{UB}}$, according to limbs’ dynamics and parameters. Therefore, $\mathbf{C}_{\text{COM}}$ results as follows:

$$-\mathbf{H}_{\text{COM}}^{\text{LB}} \leq \mathbf{C}_{\text{COM}} = -(r_{\text{EXP}} - r_{\text{COM}}) \times \mathbf{GRF} - \mathbf{M}_{\text{REF}}^{\text{UB}} \leq -\mathbf{H}_{\text{COM}}^{\text{UB}}$$

(5)

Depending on the given problem (i.e., available inputs and desired outputs), $\mathbf{C}_{\text{COM}}$ can be implemented as an unknown function evaluated through constrained optimization or a given control input in the COM dynamics.

3 Optimization Formulation

The balanced state boundary in Cartesian state space is the set of lower and upper bounds of the COM initial velocity, for each given initial COM position, that allow the system to end up in a balanced state [1]. The boundary is found by solving, with sequential quadratic programming method, the following six sets of optimization problems:

Minimize $\dot{x}_{\text{COM}}(t_{\text{initial}})$, $\dot{y}_{\text{COM}}(t_{\text{initial}})$, $\dot{z}_{\text{COM}}(t_{\text{initial}})$

(6)

Maximize $\dot{x}_{\text{COM}}(t_{\text{initial}})$, $\dot{y}_{\text{COM}}(t_{\text{initial}})$, $\dot{z}_{\text{COM}}(t_{\text{initial}})$

(7)

for all feasible COM initial position $r_{\text{COM}}(t_{\text{initial}})$, assigned though an iterative process as a partition of the COM range of motion. Other than the COM velocity extrema, each optimization loop solve for joint dynamics (modeled with B-spline), GRF and GRM (calculated through inverse dynamics), and $\mathbf{C}_{\text{COM}}$, whose discrete values at each time step are introduced as unknowns in the reduced-order dynamic model (Eq. 4). The nonlinear constraints are: a) initial COM position, b) COM Cartesian range of motion (mapped from biped’s joint angle limits), c) COP range of motion (i.e., FSR boundary functions), d) $\mathbf{C}_{\text{COM}}$ constraint and bounds (Eq. 5), e) unilateral contact with friction (i.e., $F_y \geq 0$ and $F_x^2 + F_y^2 - \mu_x F_x^2 \leq 0$), and f) final static equilibrium. The conditions of final static equilibrium are:

$$\textbf{x}_{\text{COM}}(t_{\text{final}}) = \textbf{x}_{\text{COM}}(t_{\text{final}}) = \textbf{0}$$

(8)

$$x_{\text{COM}}(t_{\text{final}}) = x_{\text{COM}}(t_{\text{final}}) \in \text{FSR}$$

(9)

$$z_{\text{COM}}(t_{\text{final}}) = z_{\text{COM}}(t_{\text{final}}) \in \text{FSR}$$

(10)

$$\gamma_{\text{COM}}(t_{\text{final}}) = \gamma$$

(11)

where $\gamma$ is an arbitrary balanced home configuration.

Any generated phase trajectory start from an initial balanced state and end at a final balanced static-equilibrium. The time duration of the generated motion $T = [t_{\text{initial}}, t_{\text{final}}]$ is selected to be large enough to allow the mechanism to reach the final static equilibrium. As a result of the iterative optimization algorithm, three balanced domain boundaries can be constructed by plotting in the $X$, $Y$, and $Z$ state spaces the resulting velocity extrema for any initial COM position assigned. This is a deterministic approach with no specific control law assumed 	extit{a priori}.

4 Results

As demonstrative results, a 4-DOF biped system [2] is mapped into the reduced-order model, which is then used to build the balanced state domain of the real system in the $X$ state space. The biped domain manifold is built for the case $\mathbf{H}_{\text{COM}} = 0$, resulting in the area that intersects the $X$ axis at the FSR constant bounds $[-0.1, 0.2]$ (Fig. 4). This is compared with the case $-100 \leq \mathbf{H}_{\text{COM}} \leq 100$ (Fig. 4), in which the larger area shows the presence of the additional multi-segmental stabilization mechanism, manifested in the reduced-order model through $\mathbf{C}_{\text{COM}}$. The velocity extrema at each position in the state space quantifies the allowable size of perturbation to maintain balance. In addition, two single support walking trajectories [2] are shown: w1 is a human-like gait and lies almost entirely in the falling state domain, showing the dynamic characteristics of human walking [2]; w2 is a robotic gait that results in a more statically balanced trajectory [2].

References
